Electromagnetic Compatibility
Problem Set 1 solutions

1.a
$$v(x+\Delta x,t)-v(x,t)=-L'\Delta x\frac{\partial}{\partial t}\left\{i(x,t)-C'\frac{\Delta x}{2}\frac{\partial v(x,t)}{\partial t}\right\}$$

$$i(x+\Delta x,t)-i(x,t)=-\left\{C'\frac{\Delta x}{2}\frac{\partial v(x,t)}{\partial t}+C'\frac{\Delta x}{2}\frac{\partial v(x+\Delta x,t)}{\partial t}\right\}$$

Dividing by Δx and taking the limit $\Delta x\to 0$:

$$\frac{\partial v(x,t)}{\partial x}=-L'\frac{\partial i(x,t)}{\partial t}$$

$$\frac{\partial i(x,t)}{\partial x}=-C'\frac{\partial v(x,t)}{\partial t}$$

1.b
$$v(x+\Delta x,t)-v(x,t)=-L'\frac{\Delta x}{2}\frac{\partial i(x,t)}{\partial t}-L'\frac{\Delta x}{2}\frac{\partial i(x+\Delta x,t)}{\partial t}$$

$$i(x+\Delta x,t)-i(x,t)=-C'\Delta x\frac{\partial}{\partial t}\left\{v(x,t)-L'\frac{\Delta x}{2}\frac{\partial i(x,t)}{\partial t}\right\}$$

Dividing by Δx and taking the limit $\Delta x\to 0$:

$$\frac{\partial v(x,t)}{\partial x}=-L'\frac{\partial i(x,t)}{\partial t}$$

$$\frac{\partial i(x,t)}{\partial x}=-C'\frac{\partial v(x,t)}{\partial t}$$

2. $f=250\ kHz,\ .Z_c=93\ \Omega,\beta=96.28\cdot 10^{-3}\ rad/m$

$Z_c=\sqrt{\frac{L'}{C'}},\beta=\omega\sqrt{L'C'}$

$\to C'=\frac{\beta}{\omega Z_c}=0.659\ nF/m$ and $L'=\frac{\beta^2}{\omega^2 C'^2}=5.7\mu H/m$

3.

$$\gamma=\sqrt{(R'+j\omega L')(G'+j\omega C')}$$

$$=\sqrt{L'\left(\frac{R'}{L'}+j\omega\right)C'\left(\frac{G'}{C'}+j\omega\right)}$$

$$=\sqrt{L'C'}\sqrt{\left(\frac{R'}{L'}+j\omega\right)\left(\frac{G'}{C'}+j\omega\right)}$$

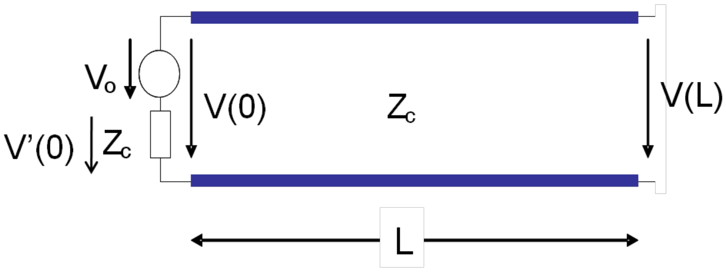
Posing $\frac{R'}{L'}=\frac{G'}{C'}=k$, the propagation constant becomes :

$$\gamma=\sqrt{L'C'}(k+j\omega)$$

With

$$\alpha=k\sqrt{L'C'}\quad,\quad\beta=\omega\sqrt{L'C'}\ \text{ and } v=\frac{\omega}{\beta}=\frac{1}{\sqrt{L'C'}}$$

4.



Inserting into the general frequency domain solutions of a two-wire transmission line the following values:

ρ_1 =0, ρ_2 =1, I_o =0, x_s =0

It results for the voltage across the left terminal impedance:

$$V'(0) = \frac{\exp(-\gamma L)}{2}[-(\exp(\gamma L) - \exp(-\gamma L))V_o] = -\frac{1}{2}[1 - \exp(-2\gamma L)]V_o$$

And the voltages $V(0)$ and $V(L)$ are given by

$$V(0) = V'(0) + V_o = \frac{V_o}{2}[1 + \exp(-2\gamma L)]$$

$$V(L) = \exp(-\gamma L)V_o$$

For a lossless line, the above equations reduce to

$$V(0) = \frac{V_o}{2}[1 + \exp(-2j\omega L/c)]$$

$$V(L) = \exp(-j\omega L/c)V_o$$

which can be converted into the time domain:

$$v(0,t) = \frac{v_o(t)}{2} + \frac{v_o(t - 2L/c)}{2}$$

$$v(L,t) = v_o(t - L/c)$$

5. a) $\rho = \frac{510-457}{510+457} = 0.055$

Transmission coefficient: $\tau = 1 + \rho = 1.055$

b) $V_{tot} = V_{inc} + V_{ref} = (1+\rho)V_{inc} = 10.55\ kV$

c) $V_{trans} = V_{tot} = (1+\rho)V_{inc} = \tau\, V_{inc} = 10.55\ kV$

$V_{ref} = \rho V_{inc} = 0.55\ kV$

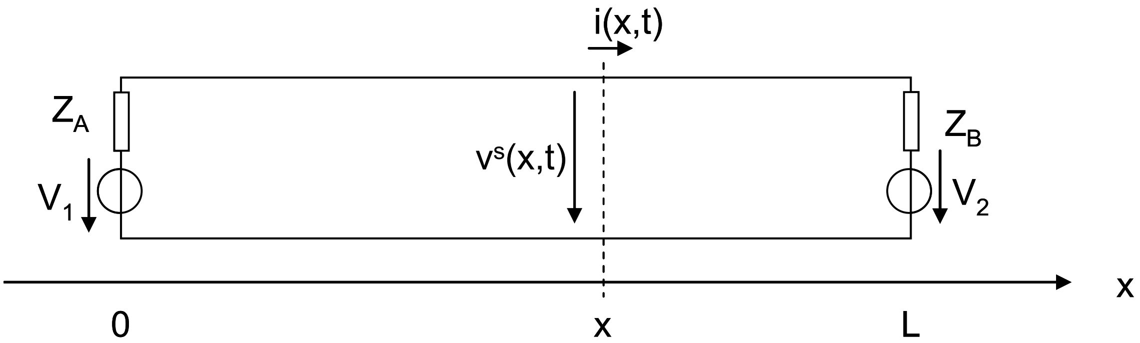
d) $I_{ref} = -\frac{V_{ref}}{Z_{c1}} = -1.2\ A$

$$I_{trans} = \frac{V_{trans}}{Z_{c2}} = 20.7\ A$$

6.

A. We choose the Agrawal et al. formulation. In this case, the coupling is represented using two lumped sources at the two line extremities.

Equivalent circuit for the coupling:



B. The expressions for the sources *V*1 and *V*2 are given by:

$$V_1(t) = \int_0^h E^e(0,t)dz = E^e(0,t)h$$

$$V_2(t) = \int_0^h E^e(L,t)dz = \int_0^h E^e(0,t)dz = E^e(0,t)h$$

The induced current at an arbitrary position *x* along the line reads :

$$i(x,t) = \frac{h}{2Z_c} E^e(0,t-x/c) - \frac{h}{2Z_c} E^e(0,t-(L-x)/c)$$

And the scattered voltage is given by:

$$v^s(x,t) = \frac{h}{2} E^e(0,t-x/c) + \frac{h}{2} E^e(0,t-(L-x)/c)$$

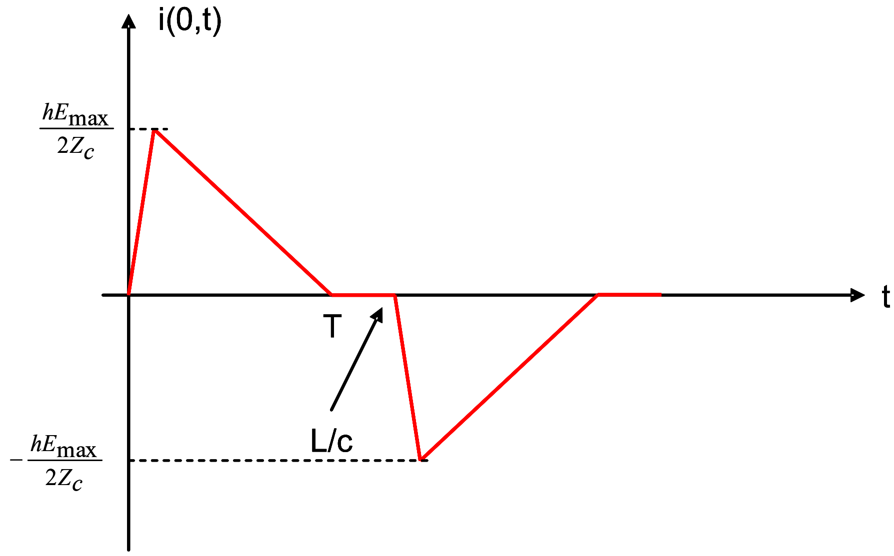
The total voltage can be obtained as follows :

$$v(x,t) = v^s(x,t) - \int_0^h E^e(x,t)dz = v^s(x,t) - E^e(x,t)h = v^s(x,t) - E^e(0,t)h$$

$$= \frac{h}{2} E^e(0,t-x/c) + \frac{h}{2} E^e(0,t-(L-x)/c) - hE^e(0,t)$$

$$= -hE^e(0,t) + \frac{h}{2} E^e(0,t-x/c) + \frac{h}{2} E^e(0,t-(L-x)/c)$$

C. Representation of the current *i*(0,*t*) for the given exciting field :



D. The currents and voltages will tend to zero.

7. Arrangement c.